AITC TECHNICAL NOTE 2

DEFLECTION OF GLUED LAMINATED TIMBER ARCHES

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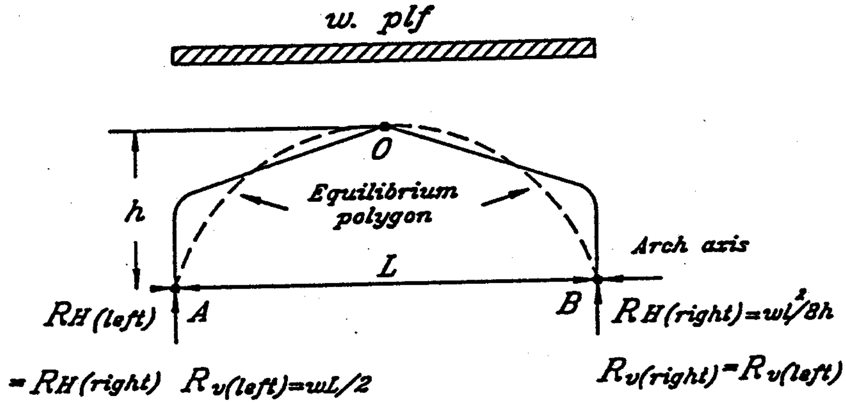


INTRODUCTION

Technical Note No. 2 has been prepared to assist the designer in calculating the deflections of glued laminated timber tudor arches. Vertical deflection at the peak and horizontal deflection at the haunch of the arch are usually calculated for dead load plus full snow load. However, other loads and load combinations, such as wind loads may be critical to the particular design and should be checked as required.

An analysis of possible arch deflections is particularly important when they are framed into glass window walls, support folding or sliding partitions, or the roof slope is nearly flat and additional deflection may cause ponding, and the designer is cautioned to account for these deflections in the building design. When glass walls frame into arches, the construction details should provide space between the arch and the glass to accommodate any movement due to arch deflections. Adequate adjustment should be provided in partition suspension systems so that adjustments can be made easily to compensate for the deflection as it occurs. Flat-topped arches should be designed and constructed so that ponding cannot occur after the arches deflect.

The designer should consult the *Timber Construction Manual (TCM)*, 4th Edition, 1994, Pages 5-268 through 5-290, for the general design procedure for three-hinged tudor arches. The principles used for calculating deflection are the same for both the mathematical solution and the graphical solution. However, the methods used to calculate the applied moments are different. While the design information presented in the *TCM* and this Technical Note only addresses symmetrical tudor arch configurations, it is noted that this type of arch is often used in unsymmetrical building plans. In many cases, the deflections, which may occur in these unsymmetrical configurations, may be more critical, and the designer is cautioned to check these.

The deflection procedure and the tabulated values shown in the *TCM* are for the mathematical solution, beginning on Page 5-290. The procedure in this Technical Note shows the tabulated values and calculations for the graphical solution for the dead load plus full snow load case. This load case was selected for illustrative purposes, but other loads and load combinations should also be checked in a complete design analysis. Also, the designer may choose to use a standard plane frame computer program to compute the arch forces and deflections resulting from the application of the required building code loadings. The arch is sub-divided into segments as in the graphical solution and the loads applied as in any plane frame analysis. Most manufacturers of glued laminated timber arches and some consulting engineering offices have developed computer programs specifically written for the analysis of the tudor arch profile.

VERTICAL DEFLECTION

The vertical deflection of arches consists of three parts:

1. Elastic or “short term” deflections due to loads.
2. Permanent set or “long term” deflections due to loads.
3. Deflection due to change of moisture content of the wood.

Elastic Deflection

The elastic deflection of an arch can be calculated by any of the usual methods of engineering mechanics. For convenience, the method of virtual work or “unit load method” is used in the design example for both vertical and horizontal deflection.

Calculations for deflections can be simplified by using the moments previously calculated for dead load (DL) plus full snow load (SL) along the length of the arch. When increased accuracy is desired, a greater number of segments can be used. Twelve segments were used in the TCM for the mathematical solution.

The procedure used in this Technical Note describes the graphical solution for vertical deflection. The arch is drawn to scale as shown in Figure 2. The scaled drawing used for determining the size of the arch may be used if it is available. The arch is then divided into segments (10 are used in this example). The segments may be of equal or unequal length, whichever is selected by the designer. If unequal lengths are used, the length of each segment is scaled from the drawing as measured along the arch axis. The equilibrium polygons are then drawn for the loading condition (DL + full SL for this example) and the unit load. For vertical loads, the moment at each point is determined by multiplying the horizontal reaction by the vertical distance between the equilibrium polygon and the arch axis at the point about which the moment is calculated. The unit load moments, m, can be determined by using the equilibrium polygon for a unit load. The unit load is placed at the location where the deflection is desired and in the direction of the assumed deflection.

The depths of the arch used for the moment of inertia calculation are also scaled form the drawing. The depths are measured along a line drawn perpendicular to the wall leg or rafter arm through the point except for points that are located in the curved portion of the arch. For these points, the depth is measured along a radial line extending from the center of curvature through the point (points C and D of Figure 2). Example 1 demonstrates the calculation of the deflection at the arch peak.

Permanent Set

The permanent set or “long term” deflection occurs in addition to the elastic deflection. Permanent set is generally more difficult to calculate, because it depends upon the moisture content of the wood and the ratio of the sustained load to the proportional limit of the wood. For glued laminated timber, the permanent set is generally considered to be about one-half of the dead load deflection. Usually this deflection is small and the inherent inaccuracies are of relatively minor consequence. Example 1 shows these typical calculations.

Deflection Caused by Changes in Moisture Content

In most cases the additional deflection caused by shrinkage does not present a problem and can be neglected. However, deflections or movement caused by changes in moisture content can be significant, particularly in design situations involving arches supporting moveable or sliding partitions, where glass walls frame into the arches or when the design involves low pitch or flat-topped arches. In these and other similar design cases where possible differential movements occurring between the arches and other framing elements are critical, the designer must make provisions to account for these movements.

The deflection caused by a change of moisture content is best explained in Fabrication and Design of Glued Laminated Wood Structural Members, U. S. Department of Agriculture, Technical Bulletin No. 1069, by Alan D. Freas and M. L. Selbo. The section regarding this subject (reproduced from pages 133-135) follows:

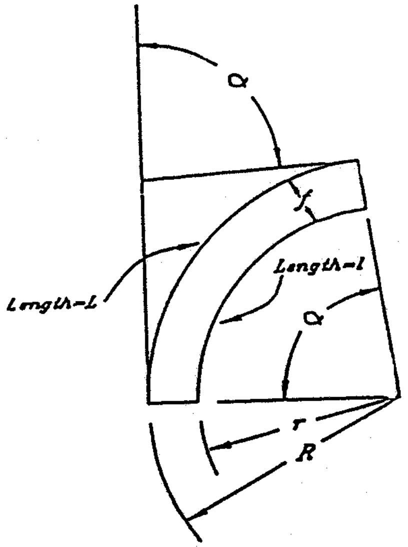
"Effect of Shrinkage or Swelling on Shape of Curved Members"

"A wood member tends to shrink or swell across the grain with loss or absorption of moisture, but practically no change occurs in the dimension along the grain. For a section of a curved wood member such as shown in Figure 67, the angle α must change to accommodate the change in thickness, since the lengths L and l do not change.

It has been shown (28) that if the thickness is changed to a thickness t (1+k), the angle α changes to a value α(1+q) where q = -k (approximately), so that the change in angle is -k α. Hence, if it is changed by a small percentage k, the angle α will be changed approximately the same percentage, but in the opposite direction; that is, radial swelling causes a decrease, and radial shrinkage causes increase, in the angle α between the ends of a curved member.

It may be noted that the percentage change in angle is independent of the length of the section, the dimensions of the cross section of the piece and the radius of curvature. The foregoing relation may therefore be used regardless of the form of the member.

In deriving this relation, an approximation was used that consisted of considering that –k/(1+k) was equal to –k, since k is small compared with unity. Slightly greater accuracy will be obtained by computing the change of angle to be –k α/(1+k).



TCM Figure 67. Diagram showing notation used in deriving the formula for change of curvature produced in a curved wood member by radial swelling or shrinkage.

It should be noted that in the application of this method several approximations cannot be avoided. These include:

(1) An average value of shrinkage must be assumed as applicable to the member. The actual shrinkage may differ considerably from the average value, and the value k will be in error by the amount of the difference.

(2) The relation used is based on the assumption that the percentage shrinkage is the same at all points. In large cross sections, the shrinkage near the outside of the cross section may be different from that in the inner part simply because of the greater time required for the moisture content to change in the inner part. Similarly, for members varying considerably in depth, a thin section will reach equilibrium throughout sooner than the thicker section; and, thus the effective value of k will be different at the two points. With sufficiently long exposure, however, and reasonably constant conditions, the value of k for all points in the member will be the same.

The effects of shrinkage or of swelling should be considered in any computation of deflection or of final position of a curved laminated member. In the case of three-hinged arches of such shape that they are horizontal, or nearly so, at the crest of a roof, such effects may be of considerable importance. In such arches there may be shrinkage enough to form a depression, or trough, at the crest of the roof that will create serious drainage problems. For an arch of this type, consideration should be given to the moisture content of the member at the time of fabrication, the moisture content to be expected in service, and the change in angle between the two ends of the member that will result from changes in moisture content and the consequent shrinkage across the grain. Consideration should be given to the provision of effective hinge details in joints of arches where such joints have been assumed in design. With such details provided, the line of action of the forces on the member will be assumed, regardless of changes in shape of the member.

Where deformation in a curved laminated member is restrained, as in a boat frame connected to a deck beam, the tendency to deform will cause stress in the member. The method just outlined will furnish the data necessary to calculate the magnitude of these stresses."

Glued laminated timbers are manufactured from lumber dried to a maximum moisture content (MC) in any piece of 16% or less. Moisture content may vary, but average moisture content is approximately 12% at time of manufacture. The in-service equilibrium moisture content of the timber is also a variable, depending upon temperature and humidity conditions. An equilibrium moisture content (EMC) of 8% is considered average for interior building uses, but can vary several percentage points higher or lower depending upon the conditions.

For calculating deflection, the designer may use the percent change q in the interior angle equal to minus the percent change in thickness k, therefore q = -k. The angle α will increase with a decrease in moisture content causing the crown of the arch to deflect downward. The value k can be determined by the use of shrinkage values from Figure 5 (also see Table 2.1, TCM) and the formula:

$S_m = S_o (I – m)/30$

where S_m =shrinkage from initial moisture condition, I, to final moisture content, m(%)

S_o =total shrinkage from Figure 5

m =final moisture content (below30%) (%)

I =initial moisture content (below30%) (%)

First, the designer should establish values for the initial MC (I) and the final MC (m). Next, a value for the total shrinkage (S_o) can be obtained from Figure 5. Figure 5 shows the linear shrinkage between 30% and 0% for Douglas Fir-Larch and Southern Pine species. Figure 5 show the different shrinkage in the tangential and radial directions, as well as the average shrinkage. Since the lumber used in the manufacture of glued laminated timber is a mixture of flat grain and vertical grain lumber, a value between tangential and radial should be considered. Generally, more flat grain lumber is used; therefore, the shrinkage value to use could be biased toward the radial shrinkage value. The following example illustrates the procedure for the calculation of elastic deflection, permanent set and deflection due to change of moisture content.

EXAMPLE 1: GRAPHICAL SOLUTION

The following example coincides with the graphical design example in the TCM. Note that points used for the deflection calculations are not the same as for the bending calculations.

Base depth = 10 ½ in.
Peak depth = 7 ¾ in.
Tangent depths = 15 ¾ in.
Arch width = 5 in.

Layout (Symmetrical)
Spacing = 15 ft.
Loads:
DL = 15 psf
SL = 25 psf
E = 1,800,000 psi

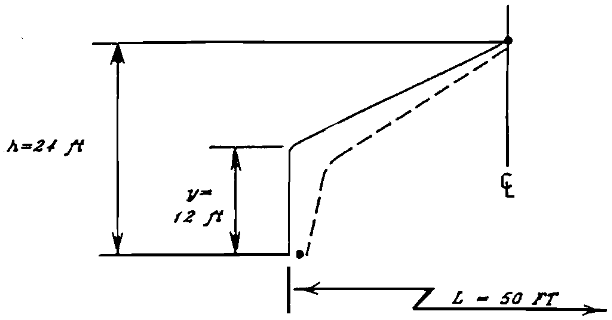


Figure 1. Basic Arch

1) Determine Elastic Deflection

The elastic deflection is based on calculations using the dimension between hinge points.

DETERMINATION OF VERTICAL (DOWNWARD) DEFLECTION OF PEAK FOR DEAD LOAD AND FULL SNOW LOAD

Point	Vertical Distance to DL + SL Equil. Poly in.	M Moment Due to DL + SL in.-lb.	Vertical Distance to Unit Load Equil. Poly in.	m Moment due to Unit Load in.	s Segment Length in.	Arch depth at point in.	I Moment of Inertia in.^{4}	Mms/I
Left								
A	-20	-153600	-22	-11.43	46	13	915.4	88190
B	-62	-476160	-65	-33.76	45	17	2047.1	353370
C	-95	-729600	-104	-54.02	45	31	12412.9	142870
D	-88	-675840	-115	-59.73	45	27	8201.3	221490
E	-55	-422400	-104	-54.02	48	17	2047.1	535000
F	-15	-115200	-82	-42.59	49	15	1406.3	170960
G	7	53760	-65	-33.76	42	14	1143.3	-66670
H	17	130560	-48	-24.93	42	12	720.0	189870
I	18	138240	-30	-15.58	45	10	416.7	-232630
J	8	61440	-10	-5.19	46	9	303.8	-48330
Right								
J’	8	61440	-10	-5.19	46	9	303.8	-48330
I’	18	138240	-30	-15.58	45	10	416.7	-232630
H’	17	130560	-48	-24.93	42	12	720.0	-189870
G’	7	53760	-65	-33.76	42	14	1143.3	-66670
F’	-15	-115200	-82	-42.59	49	15	1406.3	170960
E’	-55	-424400	-104	-54.02	48	17	2047.1	535000
D’	-88	-675840	-115	-59.73	45	27	8201.3	221490
C’	-95	-729600	-104	-54.02	45	31	12412.9	142870
B’	-62	-476160	-65	-33.76	45	17	2047.1	353370
A’	-20	-153600	-22	-11.43	46	13	915.4	88190
						Σ Mms/I = 1,948,760		

For unit load:

Horizontal reaction = 0.519 based on dimensions to hinge points

For DL + Full SL:

Horizontal reaction = 7,680 lb

Therefore, $\Delta_c = (1/E)(\Sigma Mms/I)$

$\Delta_c = 1.08$ in. or approximately 1 1/16 in.

2) Determine Permanent Set

The deflection for DL + Full SL = 1.08 in. as calculated above.

Then:

Dead load deflection = [DL/(DL+SL)](1.08) = 0.40 in.

Permanent set = Δ_{cl} = 0.5(DL deflection) = 0.5(0.40) = 0.20 in. or approximately 3/16 in.

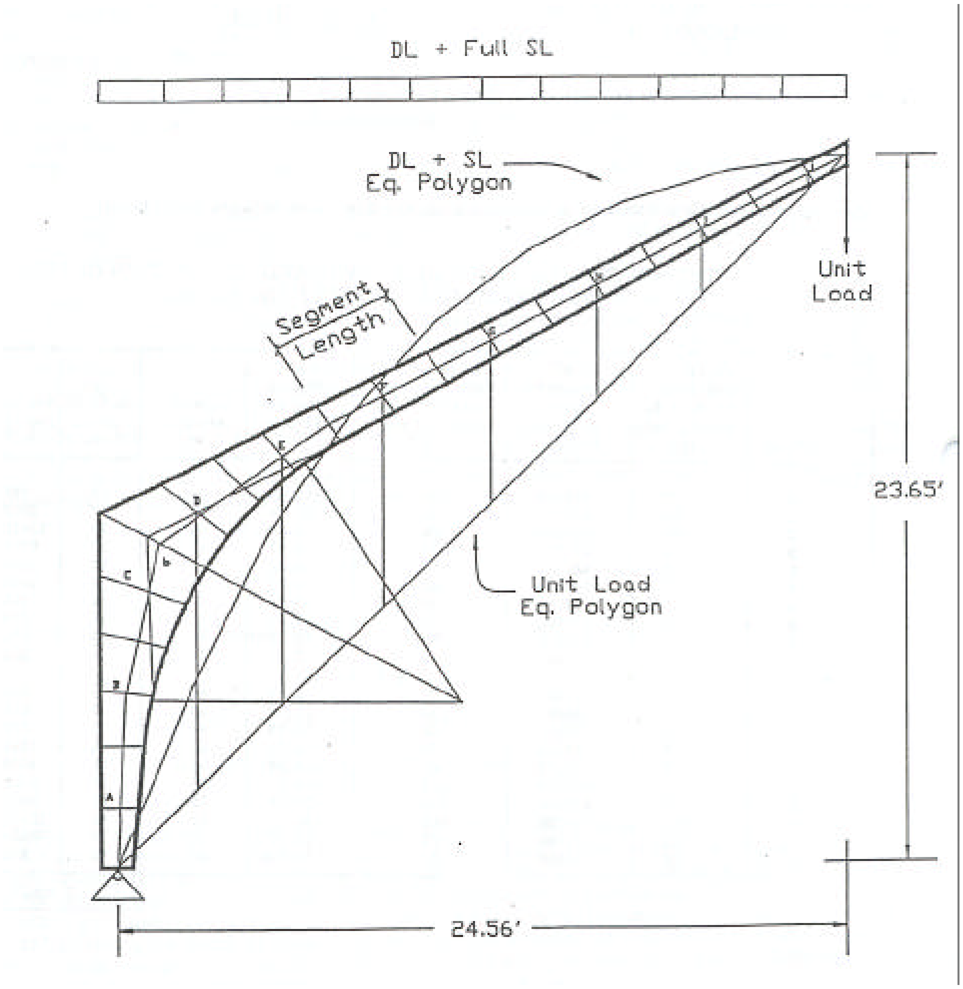


Figure 2. Equilibrium Polygons and Segments

3) Change of Moisture Content

Assume the average moisture content at the time of installation of the arch was 12% and the equilibrium moisture content is 7%. From the graph, $_{So}$ = 6.3 (average) and

S_m = 6.3(12-7)/30 = 1% and k = -S_m = -1%

An alternative solution is to use Figure 5 and determine the shrinkage values for the moisture conditions encountered. Using the average line, the shrinkage from 30% to 12% is 3.8% and from 30% to 7% is 4.8%. The difference is 4.8 – 3.8 = 1.0%. In view of the variation in shrinkage rates of individual pieces and the uncertainty of grain orientation, the ratio of 0.2% shrinkage for each 1% change in moisture content has become a generally accepted value for estimating shrinkage in both Southern Pine and Douglas Fir. The difference in shrinkage between Douglas Fir-Larch and Southern Pine is minimal as indicated by Figure 5. Therefore, k can also be determined by the following:

k = -0.2 (I-m)

Now that the shrinkage has been determined, the change in the interior angle can be calculated. As shown before, the change in interior angle α is:

Change in angle = $q\alpha = -k\alpha$

The interior angle from the example is approximately 60°. (The mathematical solution calculates this angle to be 56.8°.) The change in angle is easily calculated; however, the effect on deflection at the peak is more difficult to calculate accurately. The procedure to determine a close approximation of this deflection, Δm, is as follows:

$q\alpha = -0.01(60^\circ) = 0.60^\circ$

Draw the arch axis to scale as shown in Figure 3 or use Figure 2, which was used for the construction of the equilibrium polygons in the design procedure. Locate the midpoint of the curved section on the arch axis, point b. Assume the center of rotation of the rafter portion of the arch is located at point b. Scaled or calculated dimensions can be used to determine the distance from the midpoint, b, to the peak, c. This distance, bc, is calculated as follows:

$bc = [\,(ec^2+eb)\,]^{1/2} = [\,(23.5 + 12.32^2)\,]^{1/2} = 26.5$ ft.

The gross movement (disregarding the opposite half of the arch) is then drawn as line cd, which is perpendicular to line bc. The length of this line is determined by geometry as follows:

$cd = bc \times \tan(q\alpha) = 26.5(12)(\tan 0.60^\circ) = 3.33$ in.

This calculation gives the arc length of cd. However, because this length is so small, it can be represented by a straight line. Line cd is generally drawn to a convenient scale and is used as a proportional representation of the deflection. A length of 0.75 in. was used to represent the 3.33 in. distance cd.

Since the peak cannot move horizontally due to the restraint of the other half of the arch, assume the base hinge, point x, is free to rotate and the deflected arch is allowed to rotate about the base so that point c′ is directly below point c. The vertical deflection due to change in moisture is represented by the line cc′ where c′ is located by drawing a line perpendicular to line xc which passes through point d. The line dc′ was drawn perpendicular to line xc rather than xd, because the distance cd is drawn on an exaggerated scale and line xc more nearly represents the true position of line xd. The short dashed line represents the deflected position of the arch due to change of moisture content. The vertical deflection is calculated by measuring the distance of line cc′, which measures 0.30 in. By proportioning, the deflection is then calculated to be:

Δ_m = [0.30/0.75] (3.33) = 1.33 in. Use 1-3/8 in.

As the slope of the rafter arm of tudor arches becomes flatter, the ratio of cc′ to cd becomes larger. Also, the interior angle of the arch becomes larger resulting in a larger value of $q\alpha$, both of which tend to increase any deflection due to a change in moisture content. For instance, if the arch in the example had a total height of 18 feet rather than 24 feet the interior angle would increase to approximately 68°, the ratio cc′/cd would increase to approximately 55% and the deflection, Δ_m, would be 1.9 in.

Deflection due to change of moisture content of other shapes of curved glued laminated timbers may be determined similarly.

The total vertical deflection of the peak of the arch for this example is:

Elastic deflection = 1 - 1/16 in.
Permanent set = 3/16 in.
Change of MC = 1 - 3/8 in.

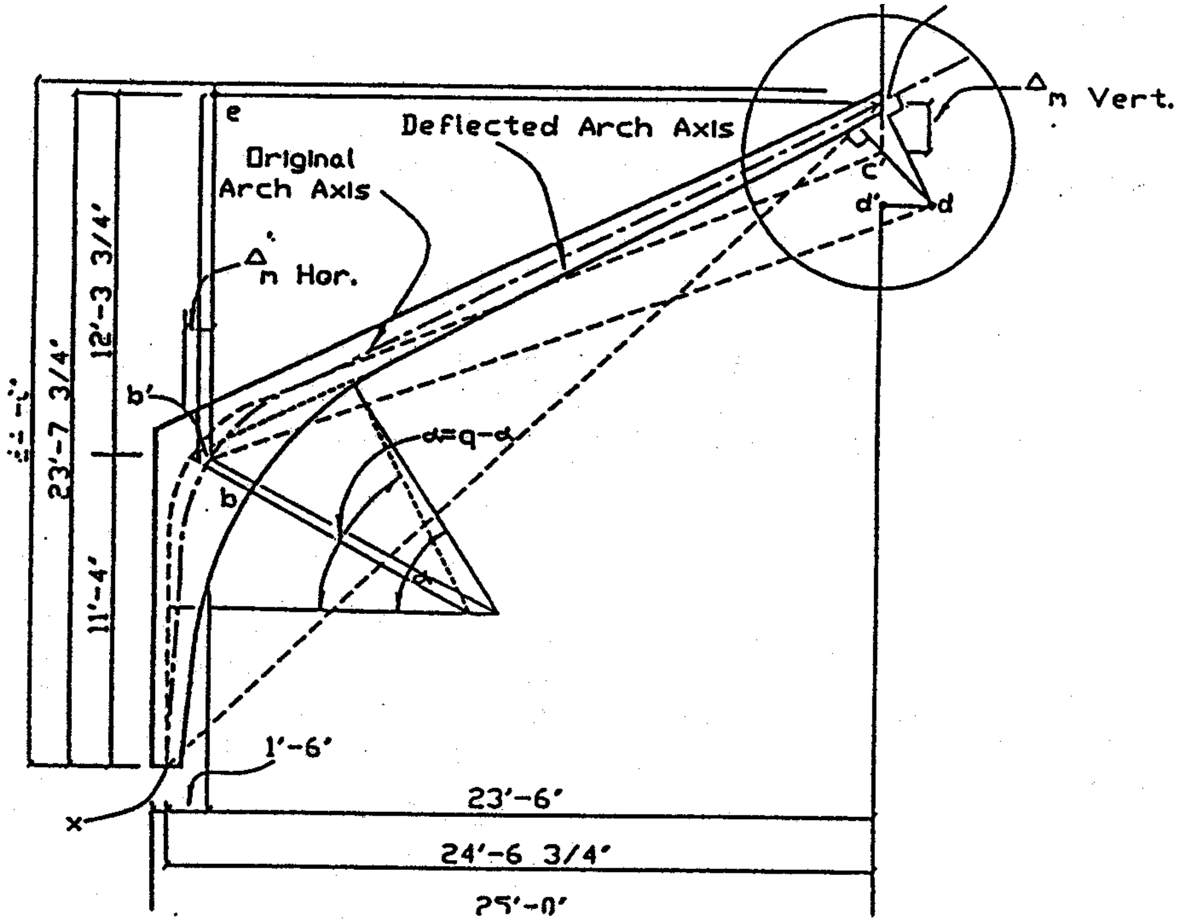


Figure 3. Arch Deflected Positions Due to Change in MC

HORIZONTAL DEFLECTION:

The horizontal deflection of arches consists of the three parts:

1. Elastic deflection due to loads.
2. Permanent set due to loads.
3. Deflection due to change of moisture content of wood.

Elastic Deflection

The horizontal elastic deflection of an arch is calculated similarly to the vertical deflection, only the unit load moment is determined by placing the unit load at the location where the deflection is desired and in the same direction as the assumed deflection. See Figure 4. The unit load moment, m, is calculated by multiplying the vertical reaction by the horizontal distance between the arch axis and the unit load equilibrium polygon. The tabulated values summarize the calculations for the horizontal deflection due to loads. Note that the distances are measured to the nearest inch, and that the final value of deflection used is to the nearest 1/16 in.

Permanent Set

Again, the permanent set horizontal deflection can be determined similarly to the vertical deflection. Generally, this deflection is very small and since the graphical solution is only approximate, the horizontal component of the permanent set is usually disregarded.

Deflection Caused by Changes in Moisture Content

The horizontal deflection due to a change in moisture content is simply calculated as shown in Figure 3. The horizontal movement of the deflected structure at the haunch is equal to the horizontal component of the line c′c (shown as d′d) multiplied by the ratio of the wall leg height to the total height of the arch.

EXAMPLE 2: GRAPHICAL SOLUTION—HORIZONTAL DEFLECTION

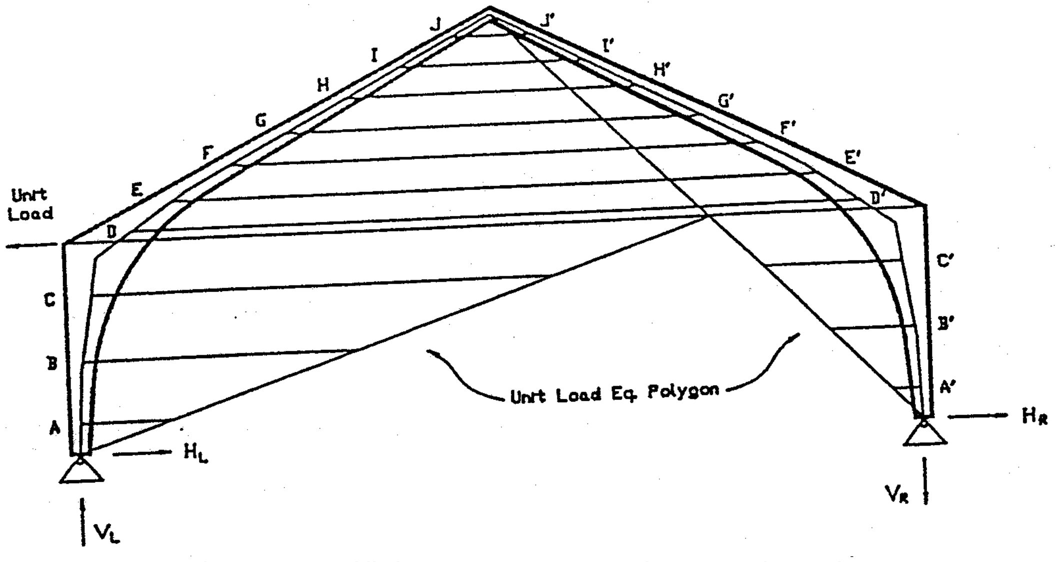


Figure 4. Equilibrium Polygons for Horizontal Unit Load

1) Determine Elastic Deflection

DETERMINATION OF HORIZONTAL DEFLECTION OF HAUNCH FOR DEAD LOAD AND FULL SNOW LOAD

Point	Vertical Distance to DL + SL Equil. Poly in.	M Moment Due to DL + SL in.-lb.	Horizonal Distance to Unit Load Equil. Poly in.	m Moment due to Unit Load in.	s Segment Length in.	Arch depth at point in.	I Moment of Inertia in.^{4}	Mms/I
Left								
A	-20	-153600	-24	-5.86	46	13	915.4	45250
B	-62	-476160	-69	-16.85	45	17	2047.1	176420
C	-95	-729600	-109	-26.63	45	31	12412.9	70130
D	-88	-675840	-119	-29.07	45	27	8201.3	107800
E	-55	-422400	-97	-23.69	48	17	2047.1	234680
F	-15	-115200	-84	-20.52	49	15	1406.3	82370
G	7	53760	-66	-16.12	42	14	1143.3	-31840
H	17	130560	-48	-11.73	42	12	720.0	-89300
I	18	138240	-30	-7.33	45	10	416.7	-109410
J	8	61440	-10	-2.44	46	9	303.8	-22730
Right								
J’	8	61440	31	7.57	46	9	303.8	70460
I’	18	138240	95	23.21	45	10	416.7	346470
H’	17	130560	153	37.37	42	12	720.0	284640
G’	7	53760	208	50.81	42	14	1143.3	100340
F’	-15	-115200	267	65.22	49	15	1406.3	-261800
E’	-55	-424400	340	83.05	48	17	2047.1	-822600
D’	-88	-675840	401	97.95	45	27	8201.3	-363240
C’	-95	-729600	340	83.05	45	31	12412.9	-217680
B’	-62	-476160	210	51.30	45	17	2047.1	-536940
A’	-20	-153600	73	17.83	46	13	915.4	-137640
						Σ Mms/I = -1076320		

For unit load:

Vertical reaction = 0.244 based on dimensions to hinge points

For DL + Full SL:

Horizontal reaction = 7,680 lb.

Therefore, $\Delta_c = (1/E)(\Sigma \text{ Mms/I})$

$\Delta_c = -0.60$ in. or approximately 5/8 in.

Negative number indicates deflection to left

1) Determine Permanent Set

Permanent set = Δ_{cl} = 0.5(DL deflection) = 0.5(15/40)(0.60) = 0.1125 in. or approximately 7/8 in.

3) Change of Moisture

Δ_m = height ratio x d′ d = 12/24(0.375/0.75)(3.33) = 0.83 in. or approximately 7/8 in.

The approximate total horizontal deflection of the haunch of the arch for this example is:

Elastic deflection = 5/8 in.
Permanent set = 1/8 in.
Change due to MC = 7/8 in.

Total horizontal deflection = 1 5/8 in.

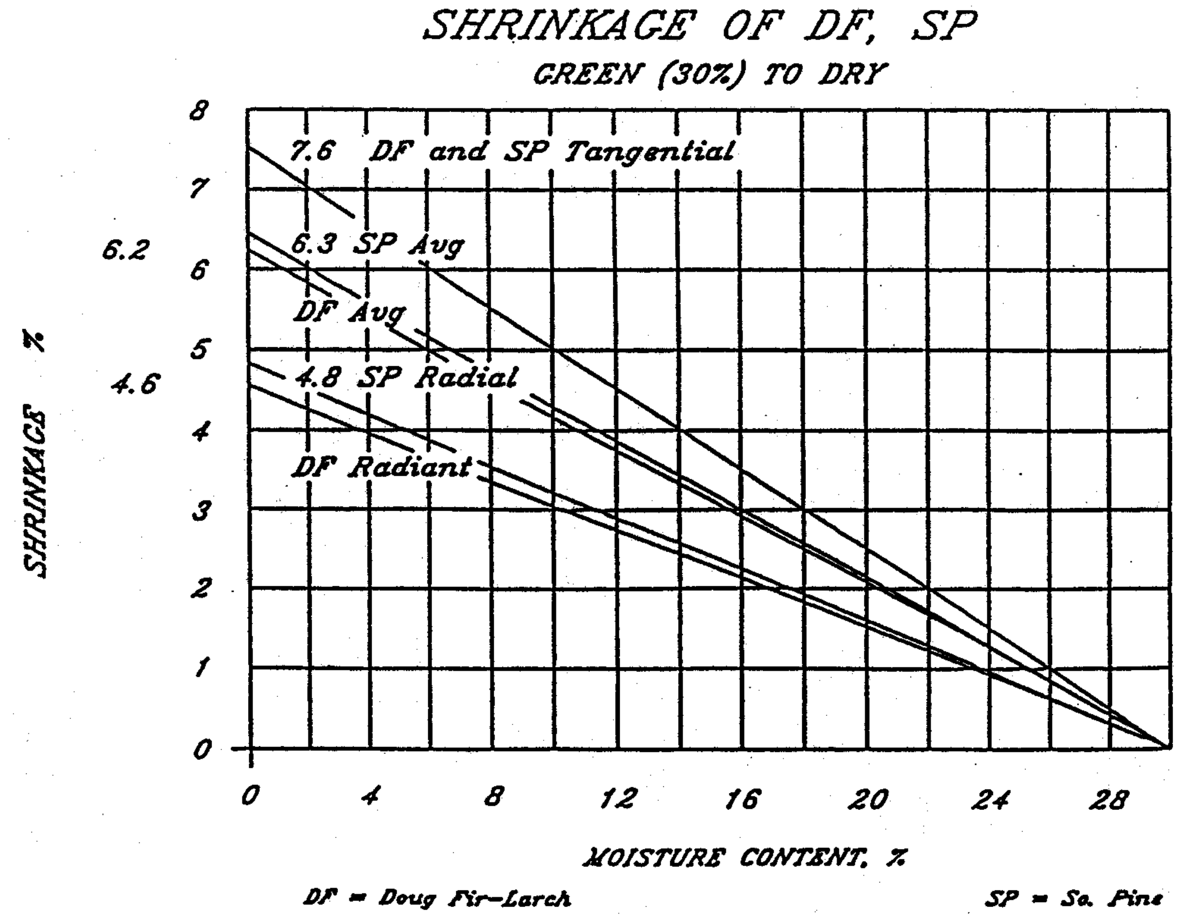


Figure 5. Shrinkage vs. Moisture content for DF and SP

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1. *Timber Construction Manual*, Fourth Edition, 1994.

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3. Technical Bulletin No. 691, *Glued Laminated Wooden Arch*, U. S. Department of Agriculture, 1939, (Out of Print)